Int. J. Therm. Sci. (2001) 40, 21–29

S1290-0729(00)01184-4/FLA

An analytical solution for a cyclic regenerator in the warm-up period in presence of an axially dispersive wave

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(Received 10 October 1999, accepted 10 February 2000)

(This paper is dedicated to Prof. K. Stephan on the occasion of his 70th birthday)

Abstract — Regenerative heat exchangers are important components of energy intensive sectors such as chemical process, power, metallurgical and cryogenic industry. The challenge of simulating regenerators accurately is considerable in view of the transient cyclic process taking place in it. The simulation of warm-up period is even more challenging due to the change in temperature profiles after each cycle. In the present work a modern technique based on the "axial dispersion model" has been utilized to simulate the regenerative heat exchanger both in the warm-up and pseudo-steady state operation. The advantage of this model is that it takes all the flow maldistribution and backmixing effects into consideration instead of idealizing the flow to be so called "plug flow". In contrast to previous studies with dispersion, in the present study the dispersion is considered to propagate with a finite propagation velocity following a hyperbolic law which is physically more consistent. The effect of different parameters on the cyclic response has been brought out and the results have been verified by comparing results of a rotary regenerator available in literature. The technique utilized in the present study can act as a tool for modelling regenerators where non-uniformity in flow distribution is significant. © 2001 Éditions scientifiques et médicales Elsevier SAS

regenerator / cyclic heating / periodic heat transfer / axial dispersion / regenerative heat exchange

Nomenclature

A	heat transfer area	m^2
A_f	free flow area (cross sectional) for the fluid	m^2
A'	variable, equation (23)	
B	heat capacity ratio $= mc_p/m_m c_{pm}$	
B'	variable, equation (23)	
C	propagation velocity of conductive wave	$m \cdot s^{-1}$
CF	complementary function, equation (25)	
C^*	propagation velocity of dispersive wave	$m \cdot s^{-1}$
c_p	fluid specific heat	$J \cdot kg^{-1} \cdot K^{-1}$
c_{pm}	matrix specific heat	$J \cdot kg^{-1} \cdot K^{-1}$
D'	variable, equation (23)	
G	variable $= p/2$	
h	heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
H	variable $= \sqrt{G^2 + q}$	
k	thermal conductivity	$W \cdot K^{-1} \cdot m^{-1}$
L	length of the equipment	m
m	mass of hold-up fluid	kg
$\dot{m}_f$	flow rate of fluid	$kg \cdot s^{-1}$
m_m	mass of the matrix	kg
M	dispersive Mach number $= u/C^*$	
N_{tu}	number of transfer units $= hA/\dot{m}_f c_p$	
$N_{tu,overall}$	overall N_{tu} given by equation (36)	
P	variable explained with equation (24)	
p	variable explained with equation (24)	
P'	variable explained with equation (27)	
Pe	dispersive Peclet number $= uL/\alpha^*$	
PI	particular integral, equation (26)	
$\dot{q}$	conductive heat flux	$W \cdot m^{-2}$
q	variable explained with equation (24)	
q_x	dispersive heat flux	$W \cdot m^{-2}$
Q	variable explained with equation (24)	
Q'	variable explained with equation (27)	
R	variable explained with equation (24)	
R'	variable explained with equation (27)	

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s	transformed frequency variable in Laplace domain	
S	variable explained with equation (24)	
S'	variable explained with equation (27)	
t	dimensionless time $= u\tau/L$	
T	temperature	K
u	fluid velocity	$\mathrm{m{\cdot}s^{-1}}$
x	dimensionless length coordinate $= X/L$	
X	space coordinate	m
Z	dimensionless blow period = blow period/residence time (τ_r)	

Greek symbols

α	thermal diffusivity	$\mathrm{m^2{\cdot}s^{-1}}$
α^*	thermal diffusivity based on axial dispersion $= \lambda/\rho c_p$	$\mathrm{m^2{\cdot}s^{-1}}$
β	variable explained with equation (32)	
γ	variable explained with equation (32)	
δ	variable explained with equation (32)	
λ	axial dispersion coefficient	$\mathrm{W{\cdot}K^{-1}{\cdot}m^{-1}}$
ξ_1, ξ_2, ξ_3	variables explained with equation (27)	
θ	nondimensional fluid temperature $= (T - T_o)/(T_{in} - T_o)$	
θ_m	nondimensional matrix temperature $= (T_m - T_o)/(T_{in} - T_o)$	
$\bar{\theta}$	θ in the Laplace domain	
ε	porosity	
ϵ	effectiveness of the regenerator	
σ	heat transfer area per unit volume	$\mathrm{m^2{\cdot}m^{-3}}$
ρ	density	$\mathrm{kg{\cdot}m^{-3}}$
τ	time	s
τ_r	residence time $= L/u$	

Subscripts

charging	during charging blow
discharging	during discharging blow
o	initial
f	fluid
in	inlet
m	matrix
$-$	just before entry
$+$	just after entry

1. INTRODUCTION

Regenerative heat exchangers are thermal energy storage devices in which heat is transferred from one fluid to the other through temporary storage in the form of sensible heat in packed beds. The bed may be packed by pebbles, metallic spheres, wire mesh, metallic wool, ceramic beads or scrap metal pieces. The primary usage of regenerators is for heat recovery in process and power industry where a very compact heat transfer equipment is needed. Gas turbine regenerators or regenerators for air liquifaction are the most compact heat exchangers available for industrial usage. Regenerators are attractive due to their simple construction, low cost, flexibility, self-cleaning operation and the wide range of temperature over which they can be used. A modern gas turbine regenerator can operate at temperatures more than 1 000 °C while a cryogenic regenerator used for helium liquifaction can operate close to 4 K. It is also interesting to note that a state-of-the-art cryogenic regenerator operates with an effectiveness of more than 99 %. In applications such as cryocoolers, the regenerator plays a key role since the performance of the whole system critically depends on the performance of the heat exchanger.

The analysis of heat transfer processes in regenerative heat exchangers are rather complex due to the fact that these equipments never work in the steady state. The principle of storing and removing heat in solid warrants for transient temperature changes in the solid as well as in the fluid. Thus, the temperature in a regenerator is always a function of time and space. This is why first regenerators were analysed in one-dimensional space coordinate only for one blow (storage blow) of the fluid considering the bed to be initially at a uniform temperature. This so-called "single blow" regenerator analysis with classical models of Anzelius [1] and Schumann [2] was further developed by many investigators such as Klinkenberg [3], Riaz [4], Montakhab [5], Vortmeyer and Schaeffer [6] and many others. Recently Adebiyi and Chenevert [7] reviewed all such models and proposed an alternative axial conductivity model. The credit of simulating thermal regenerators with both the storage and recovery blows (i.e. cyclic operation) goes to Hausen [8] which was subsequently followed by a large number of works such as Nahavandi and Weinstein [9], Bahnke and Howard [10], Heggs and Carpenter [11], Romie [12], Skiepko [13] and so on. A comprehensive overview of regenerator simulation has been presented by Schmidt and Willmott [14].

In heat exchanger analysis most of the design charts (such as LMTD-F or ϵ–N_{tu}) are constructed on the basis of an assumption of one-dimensional flow in each stream (except the case of unmixed cross flow). This type of assumption is often called "plug flow" assumption. It should be noted here that all the above works assume this a plug flow of the fluid which is a poor approximation of the actual flow process. The backmixing, leakage, recirculation, bypass, maldistribution and dead zone in the flow make the flow and heat transfer process considerably different from the assumed plug flow. To be precise, the actual detailed three-dimensional flow pattern which deviates from the conventional one-dimensional "plug flow" assumption causes the axial dispersion. In other words, from thermal effect point of view

plug flow + axial dispersion

= 3D flow with backmixing, maldistribution etc.

The effort to capture all these non-idealities is expensive even with the present day high-speed computing. An alternative approach was proposed by Taylor [15] while studying mass transfer in a conduit. He found that the concentration profile gets "axially dispersed" due to deviation from plug flow. Later Levenspiel [16] applied the same axial dispersion concept to simulate regenerator bed. The axial dispersion is proposed to be represented by a Fourier-like diffusive flux

$$q_x = -\lambda \nabla T_{\mathrm{f}} \tag{1}$$

which results in a parabolic temperature equation in the one-dimensional case, as

$$\frac{\partial T_{\mathrm{f}}}{\partial \tau} = \alpha^* \frac{\partial^2 T_{\mathrm{f}}}{\partial X^2} \tag{2}$$

It should be noted here that the axial dispersion coefficient λ is different from the fluid's thermal conductivity because it arises out of flow non-idealities. Hence, λ is a flow property and not a fluid property. A number of studies have been made in the recent past by the present author (Das) [17–19] to model heat exchangers using the axial dispersion model. Nair et al. [20] simulated a cyclic regenerator using the concept of axial dispersion.

The diffusive type of equation (1) assumes an infinite propagation velocity of dispersive wave. For conduction it has been found that the diffusive Fourier equation breaks down for very low temperature (cryogenic range) or high temperature for extremely short duration and the so-called "hyperbolic" or "non-Fourier" equation proposed by Chester [21] has to be used in the form

$$\dot{q} + \frac{\alpha}{C^2} \frac{\partial \dot{q}}{\partial \tau} = -k \nabla T \tag{3}$$

This results in a temperature equation of the form

$$\frac{\partial T}{\partial \tau} + \frac{\alpha}{C^2} \frac{\partial^2 T}{\partial \tau^2} = \alpha \frac{\partial^2 T}{\partial X^2} \tag{4}$$

Roetzel and Das [22] argued that since axial dispersion is a physical disturbance unlike molecular vibration (as in the case of heat conduction), it is likely to propagate with finite velocity even at normal temperature. Hence, it should be presented by an equation analogous to equation (3) in the form

$$q_x + \frac{\alpha^*}{C^{*2}} \frac{\partial q_x}{\partial \tau} = -\lambda \nabla T \tag{5}$$

Simultaneously Westerterp and his coworkers [23–25] developed and analyzed a similar "wave model" independently. They applied the concept to packed bed chemical reactors for both heat and mass transfer and also in presence of chemical reactions of various orders. Beneker [26] provides an extensive overview of these investigations. Recently the applicability of this "hyperbolic dispersion model" has been proven by experimental evidence in the single-blow packed bed regenerator with stainless steel wire mesh packing [27].

Thus, from the above studies it is obvious that the dispersion model in general and the "hyperbolic" (or "wave") dispersion model in particular has been established by different investigators through comprehensive analysis and experiments as a powerful tool for the analysis of packed bed heat and mass exchangers. Since the hyperbolic effect is more prominent in transient responses, the regenerator seems to be a fit candidate for the application of this model. The motive of the present paper is to simulate a cyclic regenerator both in the so-called "pseudo-steady state" and "warm-up period" using the hyperbolic dispersion model and to observe what role the dispersion and its propagation velocity plays on the overall effectiveness of the regenerator.

2. THE MATHEMATICAL MODEL

For modelling of a cyclic regenerator we take the fluid control volume and the storage material control volume from *figure 1* to arrive at the governing equation through energy balance over such control volumes. To do this we are to resort to some assumptions which are stated below:

(1) The thermophysical properties of the fluid as well as the solid are considered to be independent of temperature.

(2) The thermal conductivity of the solid is zero in axial direction and infinite in radial direction giving a one-dimensional temperature distribution in solid.

(3) The apparatus is laterally insulated from ambient.

(4) The mass flow rate is constant in each blow but it can be different for storage and recovery blows.

(5) The convective heat transfer coefficient is constant over a blow period but may be different for storage and recovery blows.

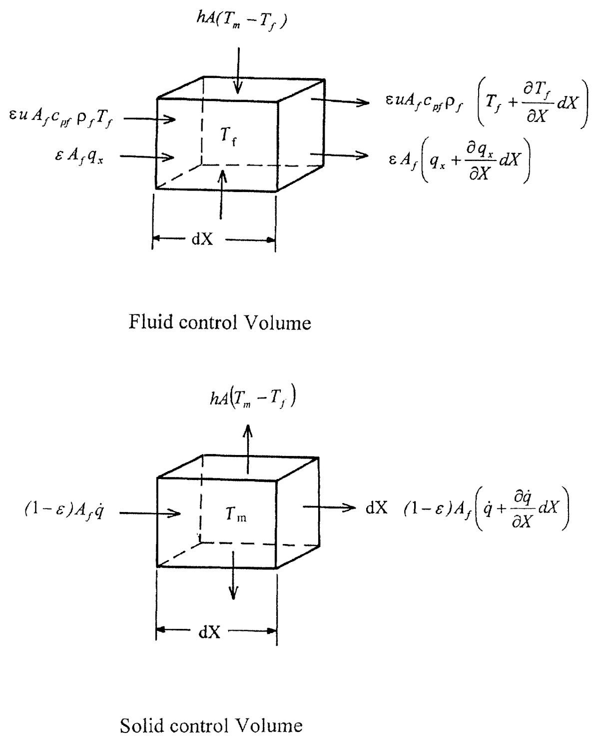

Figure 1. Fluid and solid control volumes for the regenerator.

(6) Molecular conductivity of the fluid is negligible compared to axial dispersion.

(7) The effect of “hold-up fluid” during change of blow is negligible.

(8) The packing solid is uniformly distributed over the entire apparatus giving a constant porosity in each cross section.

(9) All the deviations from plug flow in the fluid can be accounted by considering an axial dispersive wave which propagates with finite velocity through the apparatus.

With these assumptions the governing equations in terms of the axially dispersive flux q_x can be written for fluid and solid as

$$A_{\mathrm{f}}\rho_{\mathrm{f}}c_{\mathrm{p}}\varepsilon\left(\frac{\partial T_{\mathrm{f}}}{\partial\tau}+u\,\frac{\partial T_{\mathrm{f}}}{\partial X}\right) = -\varepsilon A_{\mathrm{f}}\,\frac{\partial q_x}{\partial X}+\frac{hA}{L}(T_{\mathrm{m}}-T_{\mathrm{f}}) \qquad (6)$$

$$(1-\varepsilon)\rho_{\mathrm{m}}c_{\mathrm{pm}}\,\frac{\partial T_{\mathrm{m}}}{\partial\tau}=h\sigma(T_{\mathrm{f}}-T_{\mathrm{m}}) \qquad (7)$$

Since the fluid moves in axial direction, the partial derivative with respect to time of the flux equation (5) should be replaced by the substantative derivative

$$\frac{D}{D\tau}=\frac{\partial}{\partial\tau}+u\,\frac{\partial}{\partial X} \qquad (8)$$

giving

$$q_x+\frac{\alpha^*}{C^{*2}}\,\frac{Dq_x}{D\tau}=-\lambda\,\frac{\partial T_{\mathrm{f}}}{\partial X} \qquad (9)$$

Eliminating q_x between equations (6) and (9) yields

$$\frac{1}{\alpha^*}\left[\frac{\partial T_{\mathrm{f}}}{\partial\tau}+u\,\frac{\partial T_{\mathrm{f}}}{\partial X}\right]+\frac{1}{C^{*2}}\left[\frac{\partial^2 T_{\mathrm{f}}}{\partial\tau^2}+2u\,\frac{\partial^2 T_{\mathrm{f}}}{\partial\tau\,\partial X} + u^2\,\frac{\partial^2 T_{\mathrm{f}}}{\partial X^2}\right]-\frac{\partial^2 T_{\mathrm{f}}}{\partial X^2} = \frac{h\sigma}{\rho_{\mathrm{f}}c_{\mathrm{p}}\alpha^*\varepsilon}\left[(T_{\mathrm{m}}-T_{\mathrm{f}})+\frac{\alpha^*}{C^{*2}}\left\{\frac{\partial(T_{\mathrm{m}}-T_{\mathrm{f}})}{\partial\tau} + u\,\frac{\partial(T_{\mathrm{m}}-T_{\mathrm{f}})}{\partial X}\right\}\right] \qquad (10)$$

The boundary condition for equations (7) and (10) can be given by extended Danckwerts [28] boundary condition, Roetzel and Das [22], as

$$T_{\mathrm{in}}^{+}-T_{\mathrm{in}}^{-}=\frac{\lambda}{u\rho_{\mathrm{f}}c_{\mathrm{p}}}\,\frac{\partial T_{\mathrm{in}}^{+}}{\partial X}-\frac{\alpha^*}{C^{*2}}\left(\frac{\partial T_{\mathrm{in}}^{+}}{\partial\tau}+u\,\frac{\partial T_{\mathrm{in}}^{+}}{\partial X}\right) \quad \text{at } X=0 \quad \text{and} \qquad (11)$$

$$\frac{\partial T_{\mathrm{f}}}{\partial X}=0 \quad \text{at } X=L \qquad (12)$$

These equations can be nondimensionalized using the conventional dimensionless parameters, such as number of transfer units N_{tu}, reduced length x and dimensionless time t.

Thus, the governing equations and the boundary conditions reduce to (dropping suffix f of fluid temperature for simplicity)

$$\left(\frac{\partial\theta}{\partial t}+\frac{\partial\theta}{\partial x}\right)+\frac{M^2}{Pe}\left(\frac{\partial^2\theta}{\partial t^2}+2\frac{\partial^2\theta}{\partial t\,\partial x}+\frac{\partial^2\theta}{\partial x^2}\right)-\frac{1}{Pe}\,\frac{\partial^2\theta}{\partial x^2} = N_{\mathrm{tu}}(\theta_{\mathrm{m}}-\theta)+\frac{M^2N_{\mathrm{tu}}}{Pe}\left[\frac{\partial(\theta_{\mathrm{m}}-\theta)}{\partial t}+\frac{\partial(\theta_{\mathrm{m}}-\theta)}{\partial x}\right] \qquad (13)$$

$$\frac{\partial\theta_{\mathrm{m}}}{\partial t}=BN_{\mathrm{tu}}(\theta-\theta_{\mathrm{m}}) \qquad (14)$$

$$\theta_{\mathrm{in}}^{+}-\theta_{\mathrm{in}}^{-}=\frac{1-M^2}{Pe}\,\frac{\partial\theta}{\partial x}-\frac{M^2}{Pe}\,\frac{\partial\theta}{\partial t} \quad \text{at } x=0 \qquad (15)$$

$$\frac{\partial\theta}{\partial x}=0 \quad \text{at } x=1 \qquad (16)$$

The initial condition depends on the blow under consideration. For starting from cold condition the initial temperature can be taken to be uniform for the first blow as in the case of a single blow regenerator, thus

$$\theta(0,x)=0, \qquad \theta_{\mathrm{m}}(0,x)=0 \tag{17}$$

However, for a subsequent blow of the cycle, the temperature distribution in the matrix and fluid at the end of the previous blow has to be taken as initial condition. In the present analysis a third-degree polynomial of the following form is taken:

$$\theta(0,x)=a_1+b_1x+c_1x^2+d_1x^3 \tag{18}$$

$$\theta_{\mathrm{m}}(0,x)=a_2+b_2x+c_2x^2+d_2x^3 \tag{19}$$

where the constants $a_1, b_1, \ldots$ and $a_2, b_2, \ldots$ are determined by fitting a curve to the temperature distribution at the end of the previous blow by least square method. Thus, equations (13) and (14) along with boundary conditions (15) and (16) and initial conditions (18) and (19) present the mathematical model for the cyclic regenerator.

3. THE METHOD OF SOLUTION

The solution of the mathematical model is quite complex. It is carried out by first transforming the governing equations into the frequency domain by Laplace transformation. This gives

$$\bar{\theta}_{\mathrm{m}}=\frac{BN_{\mathrm{tu}}}{s+BN_{\mathrm{tu}}}\bar{\theta}+\frac{1}{s+BN_{\mathrm{tu}}}\theta_{\mathrm{m}}(0) \tag{20}$$

This can be replaced in the Laplace-transformed fluid equations to give

$$\frac{1-M^2}{Pe}\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2}-\left[1+\frac{sM^2}{Pe}\left(2+\frac{N_{\mathrm{tu}}}{s+BN_{\mathrm{tu}}}\right)\right]\bar{\theta} = A'\frac{\mathrm{d}\theta(0)}{\mathrm{d}x}+B'\theta(0)+C'\theta_{\mathrm{m}}(0)+D'\frac{\mathrm{d}\theta_{\mathrm{m}}(0)}{\mathrm{d}x} \tag{21}$$

where $\theta(0)$ and $\theta_{\mathrm{m}}(0)$ are the temperature distributions at the beginning of the blow given by equations (18) and (19), also

$$\begin{aligned} A'&=\frac{2M^2}{Pe}\\ B'&=\frac{sM^2}{Pe}+\left[1+\frac{M^2N_{\mathrm{tu}}}{Pe(s+BN_{\mathrm{tu}})}\right]\\ C'&=\frac{N_{\mathrm{tu}}(Pe-BM^2N_{\mathrm{tu}})}{Pe(s+BN_{\mathrm{tu}})}\\ D'&=\frac{M^2N_{\mathrm{tu}}}{Pe(s+BN_{\mathrm{tu}})}\end{aligned} \tag{22}$$

Replacing $\theta(0)$ and $\theta_{\mathrm{m}}(0)$ from equations (18) and (19) gives

$$\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2}-p\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}x}-q\bar{\theta} = \frac{Pe}{M^2-1}\left[P+Qx+Rx^2+Sx^3\right] \tag{23}$$

where

$$\begin{aligned} p&=\frac{Pe}{1-M^2}\left[1+\frac{sM^2}{Pe}\left(2+\frac{N_{\mathrm{tu}}}{s+BN_{\mathrm{tu}}}\right)\right]\\ q&=\frac{s\,Pe}{1-M^2}\left(1+\frac{sM^2}{Pe}\right)\left(1+\frac{N_{\mathrm{tu}}}{s+B}\right)\\ P&=A'b_1+B'a_1+C'a_2+D'b_2\\ Q&=2A'C_1+B'b_1+C'b_2+2D'c_2\\ R&=3Ad_1+B'c_1+C'c_2+3Dd_2\\ S&=B'd_1+C'd_2\end{aligned}$$

Equation (23) can be solved by splitting it to complementary function and particular integral

$$CF=C_1\,\mathrm{e}^{(G+H)x}+C_2\,\mathrm{e}^{(G-H)x} \tag{24}$$

$$PI=\frac{1}{(D^2-pD-q)}\frac{Pe}{M^2-1}\cdot\left(P+Qx+Rx^2+Sx^3\right) \tag{25}$$

where

$$G=\frac{p}{2}, \qquad H=\sqrt{G^2+q}, \qquad D^n=\frac{\mathrm{d}^n}{\mathrm{d}x^n}$$

The term $(D^2-PD-q)^{-1}$ can be binomially expanded to give

$$PI=P'+Q'x+R'x^2+S'x^3 \tag{26}$$

where

$$\begin{aligned} P'&=\frac{Pe}{q(1-M^2)}(P+\xi_1Q+2\xi_2R+6\xi_3S)\\ Q'&=\frac{Pe}{q(1-M^2)}(Q+2\xi_1R+6\xi_2S)\\ R'&=\frac{Pe}{q(1-M^2)}(R+3\xi_1S)\\ S'&=\frac{Pe}{q(1-M^2)}S\end{aligned}$$

while

$$\xi_1=-\frac{p}{q}$$

$$\xi_2=\frac{1}{p}+\frac{p^2}{q^2}$$

$$\xi_3 = \frac{2p}{p} - \frac{p^3}{q^3}$$

Thus, the final solution in Laplace domain can be expressed as the combination of the *CF* and *PI* as

$$\bar{\theta} = C_1\,e^{(G+H)x} + C_2\,e^{(G-H)x} + P' + Q'x + R'x^2 + S'x^3 \tag{27}$$

For the determination of the constants C_1 and C_2 we have to use the boundary conditions (15) and (16) transformed into Laplace domain as

$$1 + \left(\frac{M^2 s}{Pe}\right)\bar{\theta} = \frac{1}{s} + \frac{1-M^2}{Pe}\frac{d\bar{\theta}}{dx} + \frac{M^2}{Pe}\theta(0) \quad \text{at } x = 0 \tag{28}$$

$$\frac{d\bar{\theta}}{dx} = 0 \quad \text{at } x = 1 \tag{29}$$

Using these two boundary conditions the constants can be evaluated as

$$C_1 = \frac{1}{\phi}\left[\frac{1}{s} - \beta\gamma + \delta\right] \tag{30}$$

and

$$C_2 = -\frac{C_1(G+H)\,e^{(G+H)}}{(G-H)\,e^{(G-H)}} - \frac{1}{(G-H)\,e^{(G-H)}}\left(Q' + 2R + 3S'\right) \tag{31}$$

where

$$\phi = \left(1 + \frac{M^2 s}{Pe}\right) - \frac{(1-M^2)(G+H)}{Pe} + \left(\frac{1-M^2}{Pe}\right)(G+H)\,e^{2D} - \left(1 + \frac{M^2 s}{Pe}\right)\left(\frac{G+H}{G-H}\right)e^{2D}$$

$$\beta = \left(\frac{1-M^2}{Pe}\right)(G-H) - \left(1 + \frac{M^2}{Pe}\right)$$

$$\gamma = \frac{Q' + 2R' + 3S'}{(G-H)\,e^{(G-H)}}$$

$$\delta = \frac{1-M^2}{Pe}Q' - \left(1 + \frac{M^2 s}{Pe}\right)P' + \frac{M^2\theta_{x=0}(0)}{Pe}$$

Equation (27) along with equations (30) and (31) complete the closed-form analytical solution to the problem in Laplace domain. It is obvious that it cannot be inverted back to time domain analytically. Hence, a numerical Laplace inversion technique has to be adopted. Several methods are available to carry out this inversion. However, the method based on Fourier series approximation suggested by Crump [29] has the distinction of being able to handle both monotoneous and periodic functions, hence, this technique has been used here. This algorithm computes a function $f(t)$ in the time domain from its image $\bar{f}(s)$ in frequency domain as

$$f(t) = \frac{e^{at}}{t}\left[\frac{1}{2}\bar{f}(a) + \mathrm{Re}\sum_{k=1}^{\infty}\bar{f}\left(a + \frac{ik\pi}{t}\right)(-1)^k\right] \tag{32}$$

where Re stands for the real part of the expression. The constant a is chosen in the range $4 < at < 5$ to make the truncation error small enough. For the transient response we require to calculate $f(t)$ at small intervals of time. The computation time can be significantly reduced by using fast Fourier transform. In such case to derive the following equation from equation (32), the substitution can be made in the form $t_n = 2nt/M$.

This yields

$$f(t_n) = \frac{e^{at_x}}{t}\left[\mathrm{Re}\sum_{k=0}^{M-1}\bar{f}\left(a + \frac{ik\pi}{t}\right)\exp\left(i\,\frac{2\pi nk}{M}\right) - \frac{1}{2}f(a)\right] \tag{33}$$

Using the function $\bar{\theta}(s)$ from equation (27) in this equation in place of $\bar{f}(s)$ the resultant temperature distribution of $\theta(t)$ can be computed.

4. RESULTS AND DISCUSSION

Using the solution technique stated in the preceding section temperature response for some typical cases of regenerator operation have been calculated. Results have been presented here keeping eye on the dispersion phenomenon which is the central theme of this paper rather than conventional parameters such as N_{tu}, heat capacity ratio B or ratio of charging and discharging period.

For the presentation of the results, apart from the dispersive Peclet number the other quantities used are

$$\text{effectiveness, } \epsilon = \frac{\text{average temperature difference of the fluids at exit}}{\text{maximum possible temperature difference in the regenerator}} \tag{34}$$

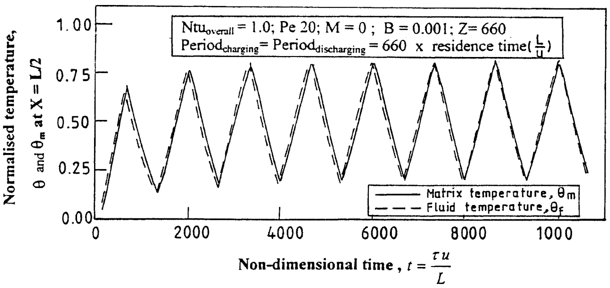

Figure 2. Mid plane fluid and matrix response during warm up and steady operation.

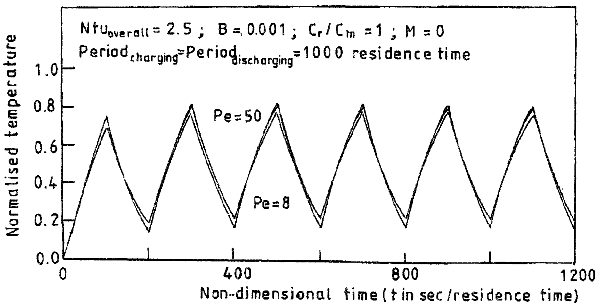

Figure 3. Effect of dispersive Peclet number on the response of the regenerator.

$$N_{\text{tu,overall}} = \left[\frac{1}{N_{\text{tu, charging}}} + \frac{(ZB)_{\text{charging}}}{(ZB)_{\text{discharging}}} \frac{1}{N_{\text{tu,discharging}}} \right]^{-1} \tag{35}$$

and

$$Z = \frac{\text{blow period}}{\text{residence time } (L/u)} \tag{36}$$

In *figures 2–4* mid plane temperatures of the regenerator have been plotted against time. *Figure 2* depicts the start-up and arrival to pseudo-steady state of a fixed bed regenerator for a particular set of operating parameters given in the legend. The usual regenerator parameters such as $N_{\text{tu, overall}}$, B and Z are chosen for ultra-compact configurations used in cryogenic industry. However, the choice of dispersive Peclet number is in order to study its arbitrary effect on the thermal behaviour in the equipment. However, a recent work [30] shows that these values lie in the realistic range. The initial temperature of the matrix and fluid is chosen to be that of the cold blow gas although this is not a limitation of the present model (any initial given temperature distribution can be used). The gradual rise of the mean operating temperature of the regenerator can be observed for the first couple of cycles.

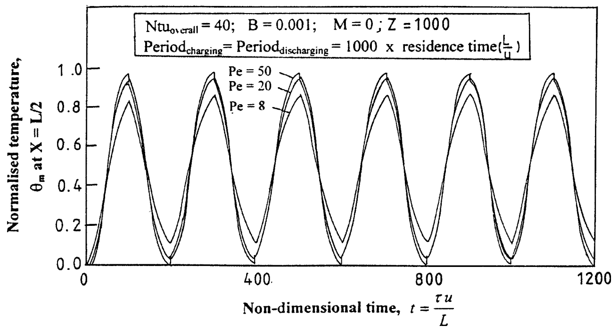

Figure 4. Effect of dispersion at high N_{tu}.

From the fourth cycle onwards the so-called “pseudo-steady state” is found to be attained. In contrast to this, in *figures 3* and *4* the pseudo-steady state was found to be attained much quicker (within two cycles) because of the fact that the cycle period in these two cases was longer ($Z = 1\,000$) than that in the previous case ($Z = 660$). The longer period gives enough time for the matrix to get heated during the warm-up blows which allows to attain the pseudo-steady state in fewer number of cycles. *Figure 2* also shows that temperature of the fluid leads the temperature of the matrix and the amplitude of temperature oscillation in the matrix is smaller than that of the fluid. Both facts are physically consistent. *Figures 3* and *4* also indicate the effect of dispersion on the temperature response. It is observed that the higher the value of dispersive *Pe*, the lower is the extent of dispersion and the larger is the amplitude of response with higher slope. This is because of the fact that at lower dispersion (higher *Pe*) lesser amount of fluid is axially mixed resulting in a flatter temperature front propagating along axial direction which has higher temperature associated with it, giving abrupt rise of solid temperature to a higher value. It can also be seen that at higher N_{tu} (*figure 4*) the decrease of thermal resistance heats the matrix to a higher value than *figure 3*. The warming up of the regenerator can be presented by the fluid and matrix temperature distribution over the entire length of the regenerator during the initial cycles. *Figure 5* represents this feature for the same data as that of *figure 3*. The data has been presented up to fifth blow because after this the pseudo-steady state is attained and the temperature profiles of the fourth and fifth blow repeat. However, the overall effect of dispersion can be understood only through the plot of effectiveness as presented in *figures 6* and *7*. *Figure 6* indicates the effect of dispersive Peclet number on regenerator effectiveness at two different blow periods. It is observed that the dispersion not only affects the thermal performance of the regenerator badly, but also the fact that the penalty increases with blow period. From the viewpoint of appli-

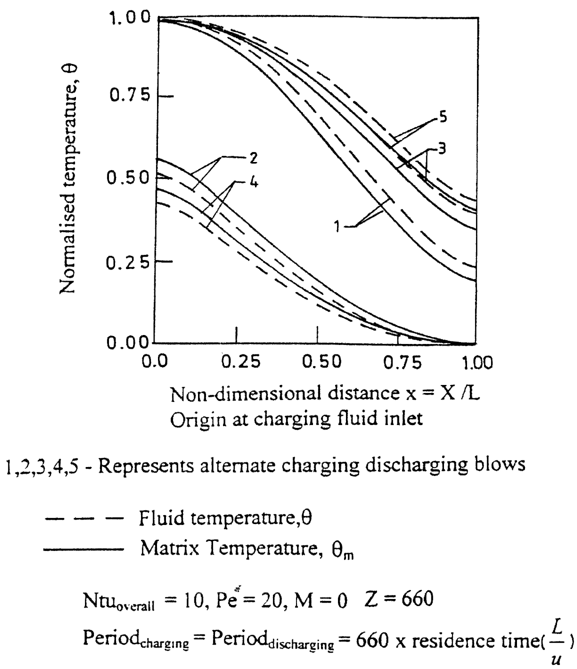

Figure 5. Axial distribution of temperature in the regenerator during successive cycles (after the fifth blow the pseudo-steady state is attained and the profiles 4 and 5 repeat).

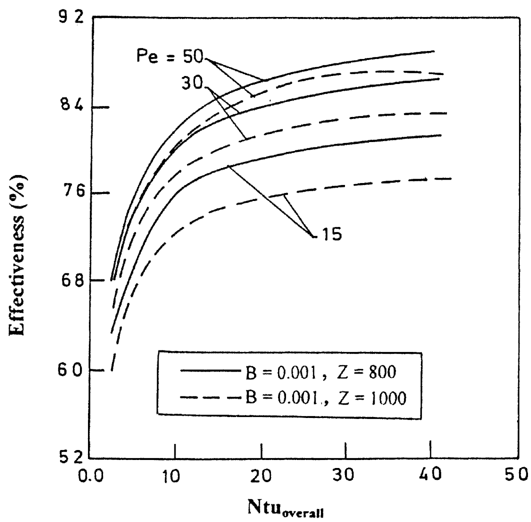

Figure 6. Effect of dispersion on the regenerator performance.

cations, this means that regenerators with high switching frequency such as cryogenic regenerators should be less influenced by the phenomenon of dispersion. It also indicates that increasing the blow period lowers the efficiency due to attainment of near-steady operation during a blow. *Figure 7* brings out the hyperbolic effect in dispersion. It is observed that the effect of propagation velocity of

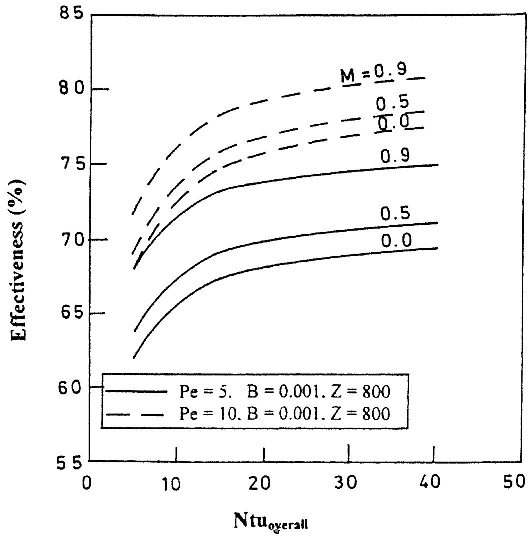

Figure 7. Effect of propagation velocity of dispersive wave on the performance of the regenerator.

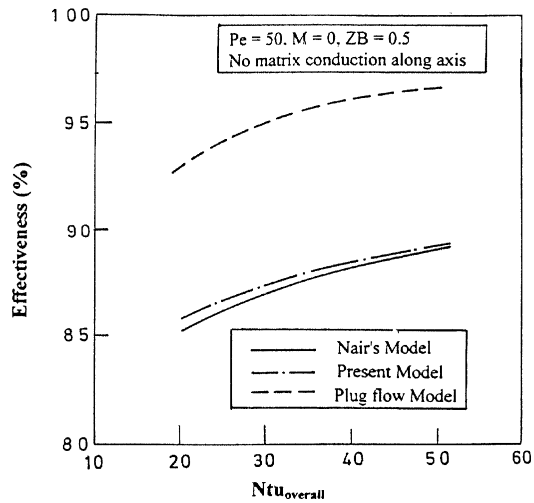

Figure 8. Comparison of the present result with the model of Nair et al. [20] and plug flow model.

dispersive wave becomes more dominant at lower Peclet number, i.e. at stronger presence of dispersion.

Finally, the results have been compared in *figure 8* to available results on rotary regenerator by Nair et al. [20] with parabolic axial dispersion ($M = 0$). It is observed that the present analytical technique matches closely to the numerical prediction and both of them vary widely from the plug flow model which justified the need for the proposition of axial dispersion. Since the present solution is the analytical one, this can be taken as exact and the deviation of [20] is due to the numerical errors.

5. CONCLUSION

A general analytical method for simulating cyclic regenerator in presence of a dispersive wave of finite propagation velocity has been presented. The model is also capable of simulating warm-up period which plays an important role in cyclic regenerator optimization. It is observed that dispersion deteriorates the performance of the regenerator considerably. It is also noted that with increasing magnitude of dispersion (i.e. decrease in *Pe*) the propagation velocity of dispersion wave starts playing increasingly significant role in the performance of the regenerator. In the special case of infinite propagation velocity of dispersive wave ($M = 0$) the model is found to agree satisfactorily with the results available in literature. While most of the available literature represent cyclic regenerators in one dimension with uniform flow distribution in pseudo-steady state, the present work brings out a comprehensive analytical technique to simulate cyclic regenerator in both steady and warm-up period in presence of flow maldistribution.

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